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A SEQUENCE OF PIECEWISE ORTHOGONAL
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Y. Y. Feng and D. X. Qi

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A SEQUENCE OF PIECEWISE ORTHOGONAL POLYNOMIALS (II)

Y. Y. Feng* and D. X. Qi**

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ABSTRACT

→ In this paper an orthonormal sequence of piecewise polynomials of degree k is given. We study the construction and sign-change properties of this sequence and consider the convergence of the corresponding Fourier series. The results generalize those obtained earlier for piecewise constant and piecewise linear functions.

AMS(MOS) Subject Classification: 41A15

Key Words: polynomial, piecewise polynomial, Legendre polynomial, series expansion, orthonormal function.

Work Unit No. 3 - Numerical Analysis and Computer Science

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SIGNIFICANCE AND EXPLANATION

We previously presented a class of piecewise linear orthonormal functions U_i that are complete in $L_2[0,1]$, and pointed out that any continuous function can be expanded in terms of U_i in the sense of uniform convergence by group. This paper generalizes those results to the case of piecewise polynomials of degree k . We construct the sequence for $k > 1$, study sign-change properties, and consider the convergence of the corresponding Fourier series. It is then shown that such a sequence of piecewise polynomials generalizes both the Walsh function and the Legendre function.

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A SEQUENCE OF PIECEWISE ORTHOGONAL POLYNOMIALS (II)

Y. Y. Feng* and D. X. Qi**

1. An Orthonormal Sequence of Piecewise Polynomials.

In this section we study a general procedure for constructing a sequence of orthonormal polynomial functions. We use the following notations:

$$Z := \{0, 1, 2, \dots\},$$

$$I_k := \{1, 2, \dots, k\},$$

$$O_n := \{1, 3, 5, \dots, 2n-1\},$$

$$E_n := \{0, 2, 4, \dots, 2n\},$$

$$\lfloor x \rfloor := \max\{n : \text{integer}, n < x\},$$

$$\langle f, g \rangle := \int_0^1 f(x)g(x)dx.$$

Suppose that $\{U_i\}_{i=0}^k$ is a sequence of orthonormal polynomials defined on $[0, 1]$, even or odd with respect to the point $x = 1/2$ and the degree of U_i is i . At first we give the following theorem.

Theorem 1. There exist exactly $k+1$ polynomials $Q_{k,i}(x)$ ($i \in I_{k+1}$) of degree k with the property that

$$U_{k,2}^{(i)}(x) := \begin{cases} Q_{k,i}(x), & 0 < x < 1/2, \\ (-1)^{k+i} Q_{k,i}(1-x), & 1/2 < x < 1, \end{cases} \quad i \in I_{k+1} \quad (1.1)$$

satisfies

$$\langle U_{k,2}^{(i)}(x), x^j \rangle = 0, \quad j \in I_k \cup \{0\}, \quad i \in I_{k+1} \quad (1.2)$$

$$\langle U_{k,2}^{(i)}(x), U_{k,2}^{(j)}(x) \rangle = \delta_{ij}, \quad i, j \in I_{k+1} \quad (1.3)$$

with

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$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

Proof. Let $k = 2m+1$ for $m \in \mathbb{Z}$. Let

$$v_{k,2}(x) := \begin{cases} Q_k(x), & 0 < x < 1/2, \\ Q_k(1-x), & 1/2 < x < 1, \end{cases}$$

$$\bar{v}_{k,2}(x) := \begin{cases} \bar{Q}_k(x), & 0 < x < 1/2, \\ -\bar{Q}_k(1-x), & 1/2 < x < 1, \end{cases}$$

where Q_k, \bar{Q}_k are polynomials of exact degree k , with leading coefficient 1. Because $v_{k,2}(x)$ is even and $\bar{v}_{k,2}(x)$ is odd with respect to $x = 1/2$, it is obvious that

$$\langle v_{k,2}, U_j \rangle = 0, \quad j \in O_{m+1}; \quad \langle \bar{v}_{k,2}, U_j \rangle = 0, \quad j \in E_m.$$

From

$$\langle v_{k,2}, U_j \rangle = 2 \int_0^{1/2} Q_k(x) U_j(x) dx = 0, \quad j \in E_m$$

we may get at least $m+1$ polynomials $Q_k(x)$, named $Q_{k,i}(x)$ ($i \in O_{m+1}$), of degree k which are linear independent in $[0, 1/2[$. The same kind of argument shows that there exist at least $m+1$ polynomials $\bar{Q}_k(x)$, named $\bar{Q}_{k,i}(x)$ ($i \in E_m$), of degree k which are linear independent in $[0, 1/2[$ and satisfy

$$\langle \bar{v}_{k,2}, U_j \rangle = 2 \int_0^{1/2} \bar{Q}_k(x) U_j(x) dx = 0, \quad j \in O_{m+1}.$$

Using the process of orthogonalization, without loss of generality, we may suppose $\sqrt{2} Q_{k,i}(x)$ ($i \in E_m$ or $i \in O_{m+1}$) are orthonormal to each other in $[0, 1/2[$, i.e.

$$\int_0^{1/2} Q_{k,i}(x) Q_{k,j}(x) dx = 1/2 \delta_{ij}, \quad i, j \in E_m \quad \text{or} \quad i, j \in O_{m+1}.$$

Let

$$U_{k,2}^{(i)} := \begin{cases} Q_{k,i}(x), & 0 < x < 1/2, \\ (-1)^{i+1} Q_{k,i}(1-x), & 1/2 < x < 1, \end{cases} \quad i \in I_{k+1}.$$

It is easy to check that $U_{k,2}^{(i)}$ satisfies (1.2) and (1.3).

Let

$$M_{2(k+1)} := \text{span}\{U_0, U_1, \dots, U_k, U_{k,2}^{(1)}, \dots, U_{k,2}^{(k+1)}\}. \quad (1.4)$$

We denote the collection of all piecewise polynomials of order $k+1$ with partition Δ_n by P_{k+1, Δ_n} , where Δ_n is the uniform partition on 2^{n-1} intervals. It is obvious that

$$\dim P_{k+1, \Delta_n} = (k+1)2^{n-1}.$$

From (1.4) we know

$$M_{2(k+1)} = P_{k+1, \Delta_2}$$

since $\dim M_{2(k+1)} = \dim P_{k+1, \Delta_2}$ and $M_{2(k+1)} \subseteq P_{k+1, \Delta_2}$. Therefore the number of polynomials $Q_{k,i}$ do no more than $k+1$. We have proved the theorem for $k = 2m+1$. When k is even, the same kind of argument confirms the theorem. |||

There are many methods for constructing the $Q_{k,i}$ and thereby the $U_{k,2}^{(i)}$ ($i \in I_{k+1}$). We now show how to do this so that $U_{k,2}^{(i)}$ satisfies some smoothness requirements at the point $x = 1/2$.

Let

$$Q_{k,i}(x) := \sum_{j=0}^k a_j^{(i)} x^j \quad (1.5)$$

on $[0, 1/2]$ with $a_k^{(i)} = 1$.

For $k = 2m$, the coefficients $a_0^{(i)}, a_1^{(i)}, \dots, a_{2m-1}^{(i)}$ ($i \in I_{2m-1}$) are defined by the following equations

$$\begin{cases} \langle U_{2m,2}^{(2i+1)}, x^j \rangle = 0, & j \in O_m, \\ \langle U_{2m,2}^{(2i+1)}, U_{2m,2}^{(j)} \rangle = 0, & j \in O_i, \quad i \in I_m \cup \{0\}, \\ \left. \frac{d^j Q_{2m,i}}{dx^j} \right|_{x=1/2} = 0, & j \in E_{m-i-1}, \end{cases} \quad (1.6)$$

with $O_0 = \emptyset$, $E_{-1} = \emptyset$,

$$\begin{cases} \langle U_{2m,2}^{(2i)}, x^j \rangle = 0, & j \in E_m, \\ \langle U_{2m,2}^{(2i)}, U_{2m,2}^{(j)} \rangle = 0, & j \in E_{i-1} \setminus \{0\}, \quad i \in I_m, \\ \left. \frac{d^j Q_{2m,2}}{dx^j} \right|_{x=1/2} = 0, & j \in O_{m-i}. \end{cases} \quad (1.7)$$

If $k = 2m+1$, the $a_0^{(i)}, a_1^{(i)}, \dots, a_{2m}^{(i)}$ ($i \in I_{2m}$) are defined by the following equations

$$\begin{cases} \langle U_{2m+1,2}^{(2i+1)}, x^j \rangle = 0, & j \in E_m, \\ \langle U_{2m+1,2}^{(2i+1)}, U_{2m+1,2}^{(j)} \rangle = 0, & j \in O_i, \quad i \in I_m \cup \{0\}, \\ \left. \frac{d^j Q_{2m+1,i}}{dx^j} \right|_{x=1/2} = 0, & j \in O_{m-i}, \end{cases} \quad (1.8)$$

$$\left\{ \begin{array}{ll} \langle U_{2m+1,2}^{(2i)}, x^j \rangle = 0, & j \in O_{m+1}, \\ \langle U_{2m+1,2}^{(2i)}, U_{2m+1,2}^{(j)} \rangle = 0, & j \in E_{i-1} \setminus \{0\}, \quad i \in I_{m+1} \\ \frac{d^j Q_{2m+1,i}}{dx^j} \Big|_{x=1/2} = 0, & j \in E_{m-i}. \end{array} \right. \quad (1.9)$$

Equation systems (1.6), (1.7) and (1.8), (1.9) define uniquely

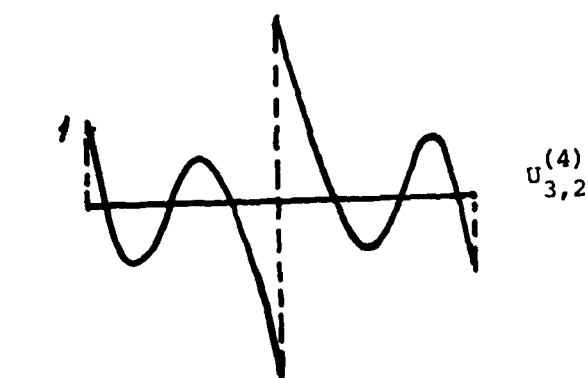
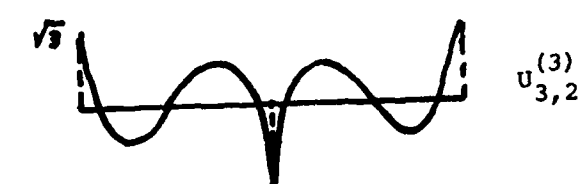
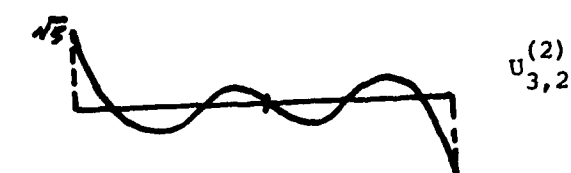
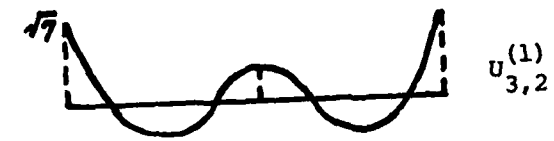
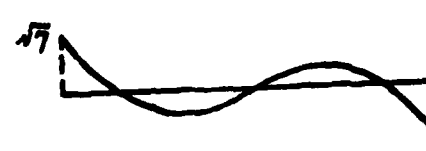
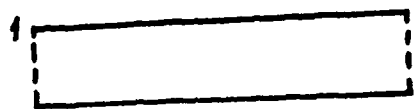
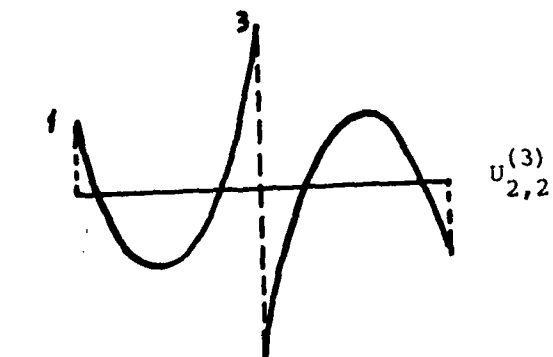
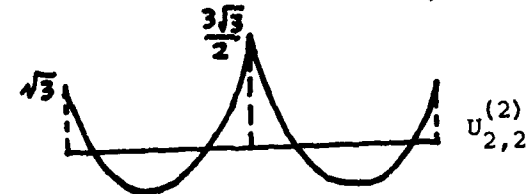
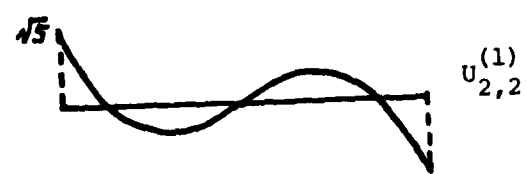
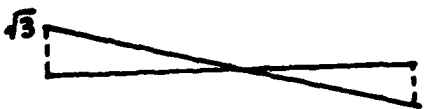
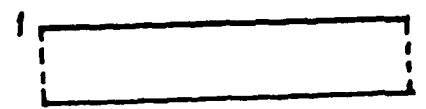
$U_{k,2}^{(i)}$ ($i \in I_{k+1}$) respectively for k even and odd. When $k=2,3$,

$U_{2,2}^{(i)}$ ($i \in I_3$) and $U_{3,2}^{(i)}$ ($i \in I_4$) are as follows after normalization:

$$\left\{ \begin{array}{l} U_{2,2}^{(1)} = \sqrt{5} (16x^2 - 10x+1), \\ U_{2,2}^{(2)} = \sqrt{3} (30x^2 - 14x+1), \\ U_{2,2}^{(3)} = 40x^2 - 16x+1; \end{array} \right.$$

$$\left\{ \begin{array}{l} U_{3,2}^{(1)} = \sqrt{7} (-64x^3 + 66x^2 - 18x+1), \\ U_{3,2}^{(2)} = \sqrt{5} (-140x^3 + 144x^2 - 24x+1), \\ U_{3,2}^{(3)} = \sqrt{3} (-224x^3 + 156x^2 - 28x+1), \\ U_{3,2}^{(4)} = -280x^3 + 180x^2 - 30x+1. \end{array} \right.$$

The graphs of these functions are given below.



After getting $U_{k,2}^{(i)}$ ($i \in I_{k+1}$), generally we define

$$U_{k,n+1}^{(2\ell-1)}(x) := \begin{cases} U_{k,n}^{(\ell)}(2x), & 0 < x < 1/2, \\ (-)^{k+\ell} U_{k,n}^{(\ell)}(2-2x), & 1/2 < x < 1, \end{cases} \quad (1.10)$$

$$U_{k,n+1}^{(2\ell)}(x) := \begin{cases} U_{k,n}^{(\ell)}(2x), & 0 < x < 1/2, \\ (-)^{k+\ell+1} U_{k,n}^{(\ell)}(2-2x), & 1/2 < x < 1, \end{cases} \quad (1.11)$$

$$\ell \in I_{2^{n-2}(k+1)}, \quad n \in \mathbb{Z} \setminus \{0,1\}.$$

We have the following theorem about the orthogonality of the sequence $\{U_{k,n}^{(i)}\}$.

Theorem 2. The sequence of functions $\{U_{k,n}^{(i)}\}$ is normal and orthogonal. I.e.

$$\langle U_{k,n}^{(i)}, U_{k,m}^{(j)} \rangle = \delta_{n,m} \delta_{i,j}$$

with $U_{k,1}^{(\ell+1)} := U_{\ell}$, $\ell \in I_k \setminus \{0\}$; $i \in I_{\mu}$, $j \in I_{\nu}$ where

$$\mu = (k+1)2^{\max(n-2,0)}, \quad \nu = (k+1)2^{\max(m-2,0)}.$$

Proof. The same kind of argument as in the proof of Theorem 1 in [3] confirms this theorem.

It is easy to see that

$$U_{k,m}^{(j)} \in \mathbb{P}_{k+1, \Delta_n}, \quad m \in I_n, \quad j \in I_{\nu}.$$

Let

$$M_{(k+1)2^{n-1}} := \text{span}(U_0, U_1, \dots, U_{k,n}^{(1)}, \dots, U_{k,n}^{((k+1)2^{n-2})}).$$

It is obvious that

$$M_{2^{n-1}(k+1)} = P_{k+1, \Delta_n}'$$

Therefore we have the following theorem.

Theorem 3. If f is a piecewise polynomial of degree k with breakpoints only at q/p , where q is integer and p is a power of two, then f can be exactly expressed by finite terms of the series

$$\sum_{i,j} a_{i,j} U_{k,i}^{(j)}.$$

2. Some Properties of the Sequence.

Let $S^+(a_0, \dots, a_n)$ denote the maximum number of sign changes in the sequence a_0, a_1, \dots, a_n obtainable by giving any zero element the value $+1$ or -1 , and define

$$S^-(f, [0, 1]) := \sup \{n : \exists t_1 < t_2 < \dots < t_{n+1}, f(t_1)f(t_{i+1}) < 0\}$$

to be the number of strong sign changes of f on $[0, 1]$.

Because $\{U_i\}$ ($i \in I_k \cup \{0\}$) is orthogonal on $[0, 1]$, it is well known that

$$Z(U_i; [0, 1]) = 1, \quad i \in I_k \cup \{0\}$$

with $Z(f; [a, b])$ denoting the number of zeros of f on $[a, b]$.

In order to study the sign changes of $U_{k,2}^{(i)}$ ($i \in I_{k+1}$) on $[0, 1]$ we need the following lemma.

Lemma 1 (de Boor[1]). If $\underline{t} = (t_i)_1^{n+k}$ is nondecreasing in $[a, b]$, with $t_i < t_{i+k}$ all i , and $f \in L_1[a, b]$ is orthogonal to $S_{k, \underline{t}}$ on $[a, b]$, then there exists $\underline{\xi} = (\xi_i)_1^{n+1}$ is strictly increasing in $[a, b]$ with $t_i < \xi_i < t_{i+k-1}$ (any equality holding iff $t_i = t_{i+k-1}$), $i \in I_{n+1}$, so that f is also orthogonal $S_{k, \underline{\xi}}$. Here $S_{k, \underline{t}}$ denotes the collection of splines of order k with knot sequence \underline{t} .

In particular, if f is continuous, then it must vanish at the n points of some strictly increasing sequence $(\eta_i)_1^n$ with $t_i < \eta_i < t_{i+k}$ all i .

It is easy to see that

$$S_{k+1, \Delta_2^{(i)}} = M_{k+1+i} = \text{span}(U_0, U_1, \dots, U_{k,2}^{(i)}),$$

where $\Delta_2^{(i)}$ is knot sequence $(t_j)_1^{2(k+1)+i}$,

$$t_j := \begin{cases} 0, & j < k+1, \\ 1/2, & k+1 < j \leq k+i+1, \\ 1, & j > k+i+1. \end{cases} \quad (2.1)$$

Using Lemma 1, we get

$$S^-(U_{k,2}^{(i+1)}) = k+1+i, \quad i \in I_k \cup \{0\}, \quad (2.2)$$

since

$$\langle U_{k,2}^{(i+1)}, S \rangle = 0, \quad S \in S_{k+1, \Delta_2^{(i)}}$$

$$\text{and } U_{k,2}^{(i+1)} \in S_{k+1, \Delta_2^{(i+1)}}.$$

We would like to study some further properties of piecewise polynomials $\{U_{k,2}^{(i)}\}$. At first, from the Budan-Fourier theorem ([4]), we know that if P is a polynomial of exact degree k , then

$$\begin{aligned} Z(P; (a, b)) &< S^-(P(a), \dots, P^{(k)}(a)) \\ &- S^+(P(b), \dots, P^{(k)}(b)). \end{aligned} \quad (2.3)$$

For convenience, suppose $k = 2m$, from (1.1), (2.2) we know

$$Z(Q_{k,i}; (0, 1/2)) = m + \left\lfloor \frac{1}{2} \right\rfloor. \quad (2.4)$$

By (1.6), (1.7)

$$S^+(Q_{k,i}(1/2), Q'_{k,i}(1/2), \dots, Q_{k,i}^{(k)}(1/2)) > m - \left\lfloor \frac{1}{2} \right\rfloor. \quad (2.5)$$

Because of (2.3), (2.4) and (2.5), we get

$$S^-(P(0), \dots, P^{(k)}(0)) = k. \quad (2.6)$$

Therefore, from Descartes' rule, we know that the coefficients of the polynomial $Q_{k,i}$ strictly alternate in sign.

A similar discussion shows that (2.6) holds when k is odd. Thus, the following lemma follows.

- Lemma 2.
1. $S^-(U_{k,2}^{(\ell)}) = k+\ell, \quad \ell \in I_{k+1},$
 2. The coefficients of the polynomial Q_{k+1} strictly alternate in sign.

By the method of construction of the sequence $\{U_{k,n}^{(i)}\}$ ((1.10), (1.11)), we know

$$S^-(U_{k,n+1}^{(2\ell-1)}) = 2 S^-(U_{k,n}^{(\ell)}),$$

$$S^-(U_{k,n+1}^{(2\ell)}) = 2 S^-(U_{k,n}^{(\ell)}) + 1,$$

thus

$$S^-(U_{k,n}^{(\ell)}) = (k+1)2^{n-2} + \ell - 1,$$

since this formula holds for $n = 2$, and follows for the general case by induction. Hence each function $U_{k,n}^{(\ell)}$ has one more sign-change than the preceding one. It is convenient to use the notation $U_{k,0}, U_{k,1}, \dots$ instead of $U_{k,n}^{(\ell)}$ when we study their sign-changes. From now on, we would like to use

both $\{U_{k,n}^{(\ell)}\}$ and $\{U_{k,n}\}$ freely with $U_{k,i} = U_i$ for $i < k$, obviously

$$U_{k,n}^{(\ell)} = U_{(k+1)2^{n-2} + \ell - 1} \quad \text{for } n \in \mathbb{Z} \setminus \{0, 1\}, \quad \ell \in I_{(k+1)2^{n-2}}. \quad (2.7)$$

Theorem 4. $S^-(U_{k,m}) = m, \quad m \in \mathbb{Z}.$

I.e.

$$S^-(U_i) = i, \quad i \in I \cup \{0\}$$

$$S^-(U_{k,n}^{(\ell)}) = (k+1)2^{n-2} + \ell - 1, \quad n \in \mathbb{Z} \setminus \{0, 1\}, \quad \ell \in I_{(k+1)2^{n-2}}.$$

Now we begin to consider the convergence properties. The Fourier series of a given function F in terms of the functions $U_{k,i}$ is

$$F(x) \sim \sum_{i=0}^{\infty} \alpha_i U_{k,i}(x) \quad (2.8)$$

with

$$\alpha_1 := \langle F(x), U_{k,1}(x) \rangle. \quad (2.9)$$

Let

$$P_n F := \sum_{i=0}^n \alpha_i U_{k,i}(x)$$

be the n -th partial sum of the series (2.8).

Then $P_n F$ is the best L_2 -approximation to F from $M_n := \text{span}(U_{k,i})_0^n$. Hence it is convergent to F if F is in L_2 , since M_n is dense in L_2 . We get the following theorem.

Theorem 5. If $f \in L_2[0,1]$, then $\lim_{n \rightarrow \infty} \|F - P_n F\|_2 = 0$.

Next we will prove that $P_{(k+1)2^{n-1}} F$ uniformly approximates $F \in L_\infty$.

It is well known [2] that

$$\|F - P_{(k+1)2^{n-1}} F\|_\infty < (1 + \|P_{(k+1)2^{n-1}}\|) \text{dist}_\infty(F, M_{(k+1)2^{n-1}})$$

and we know

$$\|P_{(k+1)2^{n-1}}\| = \|P_k\| < \infty,$$

since least-square approximation for $M_{(k+1)2^{n-1}} = P_{k+1, \Delta_n}$ is local and $M_{(k+1)2^{n-1}}$ is dense in L_∞ .

Theorem 6. Let $F \in C[0,1]$. $P_{(k+1)2^{n-1}}$ be L_2 -projector onto $M_{(k+1)2^{n-1}}$ on $C[0,1]$, then

$$\lim_{n \rightarrow \infty} \|F - P_{(k+1)2^{n-1}} F\|_\infty = 0.$$

But not every continuous function can be expanded in terms of the sequence U . We can prove that there exists a continuous function whose expansion in terms of the U 's does not converge at a point of the interval.

The same kind of argument as in the proof of Theorem 7 in [3] shows that the following theorem holds.

Theorem 7. There exists a continuous function $f \in C[0,1]$ whose expansion

$$\sum_{i=0}^n \langle f(x), U_{k,i}(x) \rangle U_{k,i}$$

in terms of $\{U_{k,i}\}$ does not converge to f uniformly when $n \rightarrow \infty$.

Acknowledgement.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this paper an orthonormal sequence of piecewise polynomials of degree k is given. We study the construction and sign-change properties of this sequence and consider the convergence of the corresponding Fourier series. The results generalize those obtained earlier for piecewise constant and piecewise linear functions.		